When does $\frac{1}{2} = \frac{1}{3}$?

modelling with wet fractions

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Many fraction activities rely on the use of area models for developing partitioning skills. These models, however, are limited in their ability to assist students to visualise a fraction of an object when the whole changes. This article describes a fraction modelling activity that requires the transfer of water from one container to another. The activity provides the opportunity for students to explore the part-whole relationship when the whole changes and respond to and reason about the question: When does $\frac{1}{2}=\frac{1}{3}$?

Although the use of concrete materials for the teaching and learning of fractions is strongly advocated, many teachers in the middle years do not use concrete materials for fraction development (Van de Walle, Karp, & Bay-Williams, 2010). The emphasis is often placed on determining the fraction of shaded areas of geometric shapes and multiple procedural computations of fractions of groups. Little attention is given to the conceptual development of fraction understanding (Mills, 2011).

Hands-on activities that are developed for students in the middle years often use the context of pizza, cookies, and food as real-life contexts and models (e.g., Bush, Karp, Popelka, & Miller Bennet, 2012; Cengiz & Rathouz, 2011; Wilson, Edgington, Nguyen, Pescocolido, & Confrey, 2011). Such activities put an emphasis of the use of the circle area model for the development of understanding of part-whole relationships, but do not address other representations such as fractions as a measure, ratio, and operator. Dominating students' experiences with the circle area model limits students' ability to transfer their knowledge of fractions to different models and contexts (Clements & McMillen, 1996). It is, therefore, important to use a variety of fraction models in order to support students to make the connections among the different fraction representations.

Mathematical models

The choice of what model to use to foster particular mathematical understanding needs to be based on the model's ability to provide links between the features of the model and the target mathematics knowledge. Stacey

and colleagues (2001) describe this as epistemic fidelity. Another factor that influences the usability of a particular model is the process of engagement students undertake with the model and is dependent on the specific socio-cultural practices of the students and established classroom practices. A third factor raised by Stacey et al. is *accessibility*. Accessibility is optimal when students "see through it [the model] to the underlying principles and relations, without being confused by features of the model itself" (Stacey, Helme, Archer, & Condon, 2001, p. 200).

The three factors described by Stacey et al. (2001)—epistemic fidelity, process of engagement, and accessibility—determine the effectiveness of concrete materials used as models. Collectively the factors contribute to the transparency of the model. Transparency is achieved when the inherent features of the model, and the way in which the model can be manipulated within particular classroom practices, supports effectively students' development of mathematical knowledge (Meira, 1998). Meira also stresses that concrete materials provide a focus for discussing mathematical ideas. In some cases the concrete materials provide vital links between the mathematics and its application in real-life contexts —an element absent in many mathematical activities.

Fraction models

Typically, three fraction models are used in the middle years of schooling—area, length, and set models. Area models help students visualise parts of the whole, length or linear models show that there are always other fractions found between two fractions, and set models show that the whole is a set of objects and subsets of the whole make up fractional parts. The three different models impart different meaning and provide different opportunities to learn. Activities designed with these models for students in the middle years mirror the way in which they are used in the primary classroom. Therefore, they have nothing more to offer the students as they progress into secondary education. The repeated use of the same models and activities in the middle years, particularly "determine the fraction of the shaded area", does not acknowledge the need to extend students' problem solving and reasoning skills as advocated by the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2015), nor does it acknowledge the need to provide older students with meaningful activities that make connections to other mathematical ideas and concepts. Also, the availability of models that provide the opportunity to explore fractions that are greater than one is limited and there is an overabundance of activities that use pie and pizza fraction models.

Alternative fraction model

In this section a fraction activity is described that introduces a different fraction model—a liquid volume model. The activity uses water in containers to explore what happens when a quantity of water is transferred into a different container, thereby giving students the opportunity to explore what happens when the whole changes in a part-whole relationship.
A collection of different sized containers is required for the activity. Odd shaped containers make it more interesting and more challenging when

visualising the fraction filled. Using a smaller container for the second part of the activity than in the first part will result in fractions greater than one. Examples of the type of containers that could be used are shown in Figure 1 and the activity is described in Figure 2.

Figure 1: Collection of containers.

When is $\frac{1}{2} = \frac{1}{3}$?

Set the scene

You have a container with water in it to water some plants. The container is not full. Your container starts to leak very slowly so you have to transfer the water into another container. First, mark the level of the water in the container. Now transfer the water into a different sized container.

Estimate

Estimate the fraction of Container 1 taken up by the water.
Estimate the fraction of Container 2 that the same quantity of water would occupy.

Measure

Use measuring cylinders to measure the volume of the water in Container 1 before transferring it to Container 2.

Record and comment

	Container 1	Container 2
Volume of container	375 mL	1.25 L
Estimation of fraction of container filled with water	$\frac{1}{3}$	
Measure of water	98 mL	
Calculated fraction of water in the containers	98375	
Express the calculated fraction as a decimal	0.26	
Percentage of container filled with water	26%	
How close was your estimate to the actual fraction for each container?		
Which container was easiest to estimate the fraction filled?		
Comment on the question: When is $\frac{1}{2}=\frac{1}{3}$?		

Figure 2: Initial investigation.

After students have conducted the initial investigation they can use the fractional quantities and the measurements made to answer questions that will assist them to develop fluency in fraction calculations. For example:

- What volume of water is required in the second container to have an equivalent fraction to that in the first container?
- How much water does each plant get if you give six plants an equal share of the water in your container?
- What fraction of the container would each of the six plants get?
- Before watering the plants you drank one fifth of the water. You then used two thirds of the water left to water three plants. What fraction of the container was used to water the plants?

Classroom discussions that occur during and after the activity can include the relationship between fractions, decimals, and percentages as well as responses to the overarching activity question. There is also the opportunity to discuss the need to convert the units of measure used. Calculations involving a 2 litre container may involve converting the container's volume of 2 litres to 2000 millimetres.

Utilisation of this activity within a sequence of learning activities designed to enhance students' understanding of fraction concepts will provide the opportunity to address multiple mathematics learning outcomes in Year 7 of the mathematics curriculum.

- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)
- Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)
- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158) (ACARA, 2015)

Conclusion

This activity takes advantage of the inherent nature of water to develop students' understanding of fractions. Because water is a liquid, it can take the shape of the container in which it is stored without changing its volume. Therefore, transferring water from one container to another allows immediate visualisation of the original quantity as a fraction within the new container. This property increases the epistemic fidelity of the fraction model underpinning the activity. The use of containers used every day by students and the familiarity students have with standard-sized drink containers increases the accessibility of the activity. There is, however, the need to explore the use of this activity further. As Meira (1998) suggests, students from different socio-cultural backgrounds may engage with this activity in unexpected ways and it is important to determine if the liquid volume model provides the transparency needed to make it an effective learning model for fraction development.

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